

# AN EFFICIENT ESTIMATOR OF MEAN WHEN POPULATION VARIANCE IS KNOWN

By

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(Received in July, 1975)

## 1. INTRODUCTION

Let  $x_1, x_2, \dots, x_n$  be a random sample of size  $n$  from a population with unknown mean  $\mu$  ( $\neq 0$ ) and variance  $\sigma^2$  (known). In order to estimate  $\mu$ , Searls (1964) has proposed a minimum mean-squared error (*m.s.e.*) estimator  $\hat{\mu} = n\bar{x}/(n + C^2)$ , where  $C (= \sigma/\mu)$  is the population coefficient of variation assumed to be known. Khan (1968) and Govindarajulu and Sahai (1972) have also utilised the knowledge of the population coefficient of variation in proposing estimators for the mean of a normal population. The purpose of this note is to propose a class of estimators for the mean  $\mu$ , when instead of the population coefficient of variation  $C$ , only the variance  $\sigma^2$  is known. In section 2 a class of estimators of the mean is proposed and its large sample properties are discussed in section 3.

## 2. ESTIMATOR

Let  $\bar{x}$  be the sample mean. Assuming that the sample mean is not near zero, we propose the following class of estimators.

$$\hat{\mu}_K = \bar{x} + K \frac{\sigma^2}{n\bar{x}}, \quad (1)$$

where  $K$  is the characterizing scalar.

If  $K=0$ , we get the usual unbiased estimator.

The bias and *m.s.e.* of the estimator (1) are

$$\text{Bias } \left( \hat{\mu}_K \right) = E \left( \hat{\mu}_K - \mu \right) = K \frac{\sigma^2}{n} E \left( \frac{1}{\bar{x}} \right), \quad (2)$$

$$\text{MSE } \left( \hat{\mu}_K \right) = E \left( \hat{\mu}_K - \mu \right)^2 = \frac{\sigma^2}{n} \left[ 2K + 1 - 2K\mu E \left( \frac{1}{\bar{x}} \right) + \frac{K^2 \sigma^2}{n} E \left( \frac{1}{\bar{x}^2} \right) \right] \quad (3)$$

The *m.s.e.* (3) is minimum when

$$K = \frac{\mu E \left( \frac{1}{\bar{x}} \right) - 1}{\frac{\sigma^2}{n} E \left( \frac{1}{\bar{x}^2} \right)} = K_{opt} \quad (\text{say}) \quad (4)$$

Then the *m.s.e.* of  $\hat{\mu}_{K_{opt}}$  is

$$\frac{\sigma^2}{n} \frac{\left[ \mu E \left( \frac{1}{\bar{x}} \right) - 1 \right]^2}{E \left( \frac{1}{\bar{x}^2} \right)} \quad (5)$$

which will always be smaller than  $\sigma^2/n$ , the variance of  $\bar{x}$ .

From (2) and (3), it may be observed that the bias and *m.s.e.* involve the expressions for the expectations of the inverse of  $\bar{x}$  and  $\bar{x}^2$  which are mathematically intractable. We have, therefore, derived large sample approximations for these expectations.

### 3. LARGE SAMPLE APPROXIMATIONS

In order to derive the large sample approximations for the bias and *m.s.e.*, we write

$$\bar{x} - \mu = t \quad (6)$$

so that

$$E(t) = 0 \text{ and } E(t^2) = \sigma^2/n. \quad (7)$$

Since the asymptotic distribution of  $\sqrt{n}t$  is normal, it can be secured that  $|P(|t| - |\mu|) - 1|$  can be made arbitrarily small if the sample size  $n$  is chosen large enough.

Employing (6) and (7), we get

$$\begin{aligned} \text{Bias} \left( \hat{\mu}_K \right) &= \frac{K\sigma^2}{n\mu} E \left[ \left( 1 + \frac{t}{\mu} \right)^{-1} \right] \\ &= \frac{K\sigma^2}{n\mu} E \left[ 1 - \frac{t}{\mu} + \frac{t^2}{\mu^2} - \frac{t^3}{\mu^3} + \dots \right] \\ &= \frac{K\sigma^2}{n\mu} \left( 1 + \frac{\sigma^2}{n\mu^2} \right), \end{aligned} \quad (8)$$

where the terms of order  $n^{-a}$  ( $a \geq 3$ ) have been neglected. Similarly to the same order of approximation, we find

$$\text{MSE} \left( \hat{\mu}_K \right) = \frac{\sigma^2}{n} \left[ 1 + K(K-2) \frac{\sigma^2}{n\mu^2} \right]. \quad (9)$$

The relative efficiency of  $\hat{\mu}_K$  w.r.t.  $\bar{x}$  is given by

$$e_k = \frac{1}{\left[ 1 + K(K-2) \frac{\sigma^2}{n\mu^2} \right]} \quad (10)$$

which is greater than one if  $0 < K < 2$ .

If we minimize the *m.s.e.* (9) w.r.t.  $K$ , we obtain

$$K = 1, \quad (11)$$

then the estimator becomes

$$\tilde{\mu} = \bar{x} + \frac{\sigma^2}{n\bar{x}}. \quad (12)$$

and its mean-squared error can be obtained from (5).

#### ACKNOWLEDGEMENT

The authors are thankful to the referee for his helpful comments.

#### SUMMARY

Under certain conditions a class of estimators for the mean of a population is proposed and its properties are studied.

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## SYMPOSIUM ON "AGRICULTURAL CENSUS, 1970-71"

The symposium on "Agricultural Census, 1970-71" was held on 26th December, 1975 at the University of Rajasthan, Jaipur during the 29th Annual Conference of the Indian Society of Agricultural Statistics. Shri I. J. Naidu, the then Additional Secretary to the Government of India presided over the symposium. Shri M.P. Rao, Joint Director (Agri. Census), Ministry of Agriculture & Irrigation was the rapporteur. Extended summaries of the papers presented and the remarks made by the Chairman and the speakers who participated in the symposium are given against the topics on which they spoke.

At the very outset the Chairman thanked the Society and all the participants for inviting him to preside over the Symposium. He mentioned that carrying out the 1970-71 Agricultural Census has been an exhilarating experience, and in its successful completion the centre had the good fortune to have the unstinted co-operation from the State/Union Territory Governments. The State Agricultural Census Commissioners and concerned technical officers took personal interest to ensure timely completion of the programme. He briefly discussed the methodology adopted.

The primary unit for data collection was the operational holding. Data were collected on the essential characteristics of agricultural holdings viz. (i) Number and size distribution of operational holdings; (ii) Land utilisation; (iii) Area under crops; (iv) Area irrigated crop-wise and source-wise; and (v) Tenancy particulars. Broadly two approaches were adopted to get the required data. In States where detailed land records exist, information required for the Agricultural Census was collected through the re-tabulation of existing information in land records. This approach was adopted in fifteen States and five Union Territories. In the States where detailed land records are not maintained, sample surveys

were organised to get the necessary data. Thus, while in the States of Kerala, Orissa, West Bengal, Meghalaya, Manipur, Nagaland and Union Territories of Arunchal Pradesh, Lakshadweep and Goa, Daman and Diu, sample surveys were organised, in the rest of the country re-tabulation method was adopted.

This census has thrown up useful data at the lower geographical units for preparation of meaningful and effective plans for agricultural development. Taking into account the experience gained so far, it was found necessary and useful to undertake the census at more frequent intervals of time. The question of carrying out the next agricultural census was examined in consultation with the States and concerned Central Ministries. It was earlier decided that the next census would be carried out on a simple survey basis with Agricultural Year 1975-76 (July 1975-June 1976) as the reference year. This decision to have 1975-76 as the reference year was later examined in the light of the Twenty Point Economic Programme, wherein great emphasis is laid on the updating of land records and implementation of land reforms. Land Records constitute the base for Agricultural Census data in most parts of the country and during 1975-76 a number of changes are likely to take place in the existing land records in view of the vigorous implementation of land reforms under the Twenty Point Economic Programme. It was, therefore, decided to postpone the reference year from 1975-76 to 1976-77.

The Chairman then pointed out the various uses to which the Census data are being put for policy formulation. He requested the participants in the Symposium to make a critical appraisal of the various aspects of last Agricultural Census and offer suggestions for improving the scope and methodology of the next Agricultural Census. Abstracts of all the 12 papers received are given below. Dr. Daroga Singh, Director I.A.R.S. summed up the discussions and thanked Shri I.J. Naidu and participants for presenting the papers and offering useful comments.

#### **Land Holdings Census, 1970-71**

P.S. Sharma<sup>1</sup>

In this paper the vital importance of Land Holdings data in the context of agricultural planning has been highlighted. It has been

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1. Jt. Director, Planning Commission. New Delhi.

pointed out that for the first time in the history of India 1970-71 Census has made available the distribution of land holdings at the district level. The availability of these data would enable to study the trends in land inequality between 1961 and 1971. This would help in identification of regions associated with high land inequality as also the regions associated with low land inequality. It would be extremely interesting to study the determinants of land inequality at the regional level. Thus it will have to be examined whether new agricultural strategy would necessarily lead to an increase in land inequality and whether the existence of pool of workers available for employment at current wages, facilitates accentuation of land inequality. It can also throw light on how the type and pattern of tenancy influences land inequality. These and certain other related hypothesis could profitably be examined in the light of districtwise data relating to land holdings.

Agricultural Census 1970-71 throws valuable light on the question whether commercial crops like cotton, oilseeds, etc., are concentrated in relatively large sized holdings. It also supplies the much needed information relating to divergence in the availability of irrigation facilities in small holdings versus large holdings. The information relating to the content of pure tenancy *viz.* whether it is operated on fixed money/kind terms or on share of produce will enable to have meaningful discussion on tenancy and productivity.

### Economic Analysis Of Agricultural Census Data

R. Padmavalli<sup>2</sup>

In this paper a study has been attempted to analyse the impact of the size of the operational holding on the cropping intensity and gross agricultural income from the yield of crops. The analysis was based on the data pertaining to Agricultural Census, 1970-71 in respect of Tamil Nadu. On analysis it was found that a mono-cropping pattern was prevalent in Thanjavur, Chingleput and North Arcot districts which would mean that their agricultural income would be greatly dependent on the farm harvest prices of a few crops

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2. Director of Statistics, Madras.

whereas much fluctuations may not be expected in cases of districts having a tendency towards multiple cropping pattern. The analysis has also revealed that the cropping intensity decreased with the increase in the size of holdings. This trend was prevalent in almost all districts of the State. The States' distribution of gross agricultural income by size of holdings revealed that the income increased from the first size class viz. 0-0.5 and reached a maximum value in size class 1-2, thereafter it decreased uniformly but for size class 5-10 and the last size class. It has been observed that the small holdings make the most use of irrigational facilities though it cannot be said the best use in the absence of input data.

#### **Methodological Aspects Of World Agricultural Census 1970-71 With Special Reference To Andhra Pradesh**

V. Raman Rao<sup>3</sup>

The analysis and tabulation of 1970-71 Agri. Census data pertaining to Andhra Pradesh had thrown up the following issues.

- (i) There was no scope to build up the information of ownership holdings.
- (ii) There were variations in the area reported under different characteristics as reported in the official Agri. Statistics 1970-71 and the data thrown by Agri. Census, 1970-71.
- (iii) Information obtained on part holdings of an operator in the other villages was incomplete and it could not be ensured fully to combine such part holdings data to form the complete holding of an individual.

These problems have been discussed in details. The areas where the Agri. Census data would be useful for policy purposes have also been discussed in this paper.

#### **Sampling For Agricultural Census—Orissa's Experience**

C. Mishra<sup>4</sup>

The details of Agricultural Census, 1970-71 which was a large scale sample survey in Orissa have been discussed. The scale of

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3. Jt. Director, Bureau of Economics & Statistics, Hyderabad.

4. Director, Bureau of Statistics & Economics, Orissa.



operation or the sampling fraction needed to obtain reliable and operationally usable results through a sampling enquiry in preference to complete enumeration has been examined. Before it is done so, one has to consider the level at which such information is needed by agricultural planners. If for agricultural census or for the collection of any agricultural statistics, revenue agency is to be employed for field work, one has to maintain the Tahsil as the stratum. The Tahsil & Block boundaries are normally not coterminous. Hence while evolving a sampling design, one has to plan the enquiry in such a way that Block level estimates could be built up, through post stratification, if necessary. The sampling design adopted in the last Agricultural Census has been discussed in this paper. Sampling errors and non-sampling errors play a prominent role in a sample survey of land holdings through enquiry. Some of these errors have been also mentioned in this paper.

#### **The Role of The National Sample Survey (NSS) In Implementing The Programmes Of The World Census Of Agriculture In India**

J. K. Aikat<sup>5</sup>

The paper gives an account as to how the FAO programme of World Census of Agriculture has been implemented in India and the special role played by the National Sample Survey Organisation towards the beginning in 1954-55 and subsequently in 1960-62 and also in 1971-72 when the All India Census of Agriculture was conducted. Some details of three extensive Land Holdings surveys of India, conducted alongwith the socio-economic surveys of the 8th, 16th, 17th, and 26th Rounds of NSS, are given here. The results of the 26th Round Land Holdings survey for some key characteristics are discussed and salient features brought out. A comparison of the results of this latest survey and those of the 1970 All India Census of Agriculture has also been made for two characters (number and area of operational holdings) and the areas of agreement and disagreement have been pointed out. The comparison has been made in respect of All India and one State, namely West Bengal, by way of illustration.

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5. Deputy Director, National Sample Survey Organisation, New Delhi

**Agricultural Census 1970 In Uttar Pradesh.**

G.C. Srivastava<sup>6</sup>

The discussion in this paper is mainly confined to the Agricultural Census 1970-71 operations in Uttar Pradesh. The organisational aspects, methodology adopted, tabulation procedures and the important results have been given. No comprehensive system of maintaining land records exists in the hill portion of Uttar Pradesh and yearly crop enumeration on field to field basis is not carried out as a result of which neither the crop-wise area nor land utilisation statistics are available on a complete enumeration basis as in the plains. One of the objectives of agricultural census was to fill in this gap. Accordingly sample census in the hill region was so designed as to provide reliable estimates at the district level for different categories of land utilisation. The sampling design adopted, the estimation procedures used and the results derived from the sample census have been discussed in detail.

Some suggestions for future sample census and uses to which the last Agricultural Census data have been put to have also been discussed.

**Agricultural Census In Karnataka 1970-71: An Economic Analysis Of Tenancy In Relation To Irrigation**

M. ahadeviah<sup>7</sup>

In this paper an attempt has been made to analyse the data on tenancy in relation to the irrigation facilities in the State of Karnataka as revealed by Agricultural census 1970-71. The State has 2.34 lakh holdings and 4.60 lakh hectares under tenant cultivation accounting for 6.6 percent of the total holdings and 4.1 percent of the total operated area of the State. The small holdings (below 2.0 hectares) claim nearly 70 percent of the holdings and 27 percent of the area under tenant cultivation in the State. Coastal region alone claims

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6. Dy. Director, Revenue Deptt., Govt. of U.P., Lucknow.

7. Dy. Director of Statistics, Agricultural Census, Karnataka,

about 47 percent of the total holdings and 2.5 percent of the area in the State under tenant cultivation while the small holdings claim 82 percent of the holdings and 51 percent of the area of the region under tenant cultivation.

About 10.7 percent of the total holdings and 4.4 percent of the net area sown in the State are wholly irrigated. The small holdings claim 82.6 percent of the holdings and 4.4 percent of the area wholly irrigated in the State. About 80 percent of the holdings under tenant cultivation in the coastal region are unirrigated while the number of holdings under tenant cultivation which are unirrigated in other regions is not quite large.

#### Sample Surveys For Agricultural Census In West Bengal

M. Betal<sup>8</sup>

The paper gives an account of the importance of agricultural statistics. The sampling design adopted and the agency used for collection of data pertaining to 1970-71 Agricultural Census in West Bengal have been discussed. The importance of accuracy of the census data has also been discussed.

#### On The Adoption Of Sample Survey Methods For Censuses Of Agriculture

N. Gopalakrishnan Nair<sup>9</sup>

Statistical data of the type collected in Agricultural Census are increasingly sought by the planners in developing countries. Besides providing micro-data on basic key items to serve the overall needs of planners and to provide an insight into the state of the agricultural economy, the Agricultural Census serves the vital purpose of providing a frame and benchmark on which to have future surveys and to produce supplementary information to improve estimation procedures. The usefulness of the census as a means of providing data for planning purposes and its potential are widely recognised. The conduct of a full scale census, however, implies large resources

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8. Dy. Director of Agricultural (Census & Enumeration), Government of West Bengal, Calcutta.

9. Director, Bureau of Economics & Statistics, Kerala, Trivandrum.

in man-power and money. It is in this situation that the use of sampling methods comes in as a practical alternative. Where funds and conditions permit, data on basic items can be collected by complete enumeration for establishing benchmark data and for other items, sophisticated methods of sampling can be adopted. However, when resources and facilities for collecting the basic items by complete enumeration are lacking and the immediate needs of planning can be met by data for larger administrative units, the complete enumeration can be replaced by a sample census. Sample census have an added advantage in that the methodology is feasible.

Special attention should be paid to the selection of suitable techniques, drafting of questionnaire, training of field staff, publicity, method of enumeration and presentation of final results. An efficient method of sampling calls for a thorough study of the situation to which it is applied. Sample Censuses are adaptable to a wide range of situation so that the samples have abundant choice depending on the type of problems and conditions.

In this paper, an attempt has been made to indicate some of the problem areas. Sufficient expertise and knowledge have accrued in the conduct of the last census and it is worthwhile to utilise the experience of the personnel engaged in the census for further studies. There is scope for substantial improvements in the methodology and working procedure used at present and this require sustained effort of all those connected with the Agricultural Census.

### **Organisation & Methodological Aspects With Reference To Madhya Pradesh**

S.C. Johri<sup>10</sup>

The organisational set up for collection of Agricultural statistics in the State of Madhya Pradesh has been discussed in this paper. The methodology adopted for the collection of data pertaining to Agricultural Census, 1970-71, the tabulation programme and the problems faced due to which the census operations were delayed have been enumerated. The utility of Census results has been discussed broadly.

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10. Land Records Department, Gwalior, Madhya Pradesh,

It has been concluded that the Agricultural Censuses needs to be given national importance like the population census. It is essential to conduct the census at periodic intervals so as to make the data available for formulation of realistic plans and policies. In this context it seems necessary to collect data on both operational as well as ownership holdings. It is worthwhile enacting an agricultural census legislation so that the census is carried out with due care and priority and the data are available for use in time.

### **Organisational Aspects Of Agricultural Census 1970-71 & Use Of Census Results For Policy Purposes With Special Reference To Bihar**

S.R. ADIGE<sup>11</sup>

The historical background, importance of Agricultural Census, organisational arrangements for 1970-71 Agri. Census, items on which data were collected, time schedule, of census operations, - updating of land records, imparting of training at different levels, tabulation procedures, publicity given, information on various problems faced etc. have been discussed in this paper.

The Agricultural Census data would facilitate a periodical evaluation of the progress of development plans and provide useful tools for land reforms particularly ceiling on land holdings, consolidation of holdings, incentives to increase investment and promotion of improved agricultural techniques. Irrigation programmes, the demand and use of fertilisers and the requirements of agricultural credit are some of the other major areas where the census data would be found useful. In addition, the special area programmes, such as SFDA, MFAL projects and tribal development projects would also find these data useful for programme formulation, implementation and evaluation. Furthermore, the agricultural census results would also provide useful source material in the national income statistics and for making further refinements therein.

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11. Dy. Director (Senior), LBS NAA, Mussoorie, U.P.

### Irrigation In Himachal Pradesh

Birpal Singh<sup>12</sup>

It has been revealed by Agricultural Census 1970-71 that kuhls are the main source of irrigation in Himachal Pradesh. Since wells and tube-wells cannot be constructed in the middle and high hills, irrigation by kuhls is the only alternative. The construction of kuhls is a labour intensive task and require comparatively less capital expenditure. Hence, farmers with higher manland ratio (Marginal and Small farmers) have taken more advantage of this source of irrigation. Therefore, the districts where kuhls are the only source, small land holders are comparatively better off than their counterpart big land holders in respect of irrigation facilities. It has, however, to be realised that in most of the areas (river valleys) gravity flow irrigation is not possible because the agricultural land are at a higher level than the source *i.e.* stream or river and lift irrigation is the only answer.

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12. Statistician, Agril. Census, Himachal Pradesh, Simla-4.